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CONCRETE MIX DESIGN AND OPTIMIZATION

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SUMMARY

Determination of ingredients of aggregate mixes is discussed as an important part of concrete mix design. Different types of “ideal” aggregate grading curves are presented in the work. Analytic and numerical methods of aggregate mix design are proposed. A concrete mix design method is used taking into account granulation parameters of aggregates. The task of concrete mix optimization implies selecting the most suitable concrete aggregates from the Data Base. The following properties are to be optimized: cost of raw materials, quality of aggregate packing, water and cement consumption. Computer programs for aggregate and concrete mix design as well as for concrete mix optimization have been worked out.

Keywords: Concrete, aggregate, grading, “ideal” curve, optimization

1. DESIGN OF AGGREGATE MIX

Aggregate takes up 60 - 90% of the total volume of concrete. Proper selection of aggregate type and particle size distribution affect the main properties of concrete – workability of concrete mix as well as mechanical strength, permeability, durability and the total cost of hardened concrete, therefore aggregate mix design is an essential part of concrete mix design and optimization. There are two ways to determine the composition of an aggregate mix: by means of an “ideal” grading curves and by means of theoretical and practical determination of aggregate packing value.

1.1 “Ideal” grading of aggregate

Aggregate grading is defined as relation between size of standard sieve X_i (mm) and the total amount passing through this sieve $Y_i(X_i)$. This relation can be reflected by formulas, tables or graphics. Optimum aggregate grading is described by means of “ideal” grading curves which provide good aggregate packing and the best properties of concrete.
There are different types of “ideal” curves worked out on the basis of practical experiments and theoretical calculations – Bolomey's, Fuller's, Graf's, Rissel's curves

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[1,2,3,4]. The most known and acceptable of them is Fullers “ideal” curve. It is described by a simple equation:

$$YT_i = 100\sqrt{X_i / X_{max}}$$

here YT_i - is “ideal” (theoretical) passing, %;
X_{max} - is aggregate maximum size (finish point of an “ideal” curve).

It must be taken into account, that usually an “ideal” curve starts at point (X_0, 0) because particles smaller X_0=0.075 mm are dust and clay. Thus, taking into account this factor, equation of an “ideal” Fuller’s curve is:

$$YT_i = T\sqrt{X_i - X_0} = T(X_i - X_0)^{0.5}$$

here T - is the coefficient, dependent on the maximum size of aggregate.

It must be mentioned, that Fuller’s curve degree can be modified depending on type of used aggregate (angular or rounded) [5]. On the other hand, Fuller’s curve gives good results, if stiff concrete mixes with low workability are used. For plastic concrete mixes (Cone Slump 5 cm and more) and especially for pumped concrete the amount of sand must be increased. For this reason it is suggested to change Fuller’s equation degree depending on consistency of concrete and type of aggregate. Thus, transformed Fuller’s curve equation is as follows:

$$YT_i = T_n(X_i - X_0)^n,$$

here n - is degree of an “ideal” curve equation;
T_n - is coefficient, dependent on maximum size of aggregate and the degree of equation.

“Ideal” grading of aggregate can also be defined by means of restricting curves in graphics [8]. Different “ideal” curves (also transformed Fuller’s curves with different degree) and restricting curves in accordance with DIN 1045 [5] are shown in Fig.1.

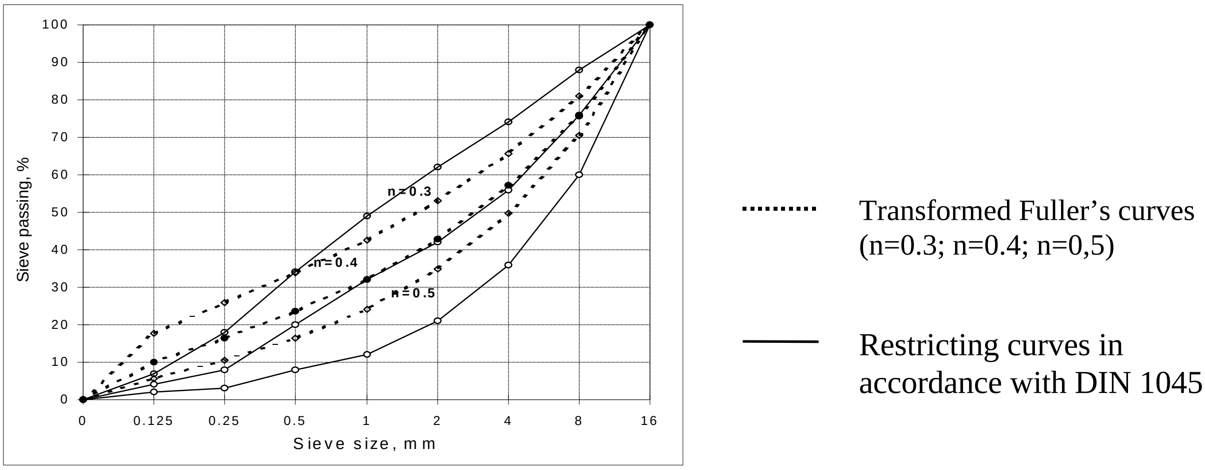

Fig.1 Aggregate “ideal” grading

“Ideal” aggregate grading can be provided if sand and course aggregate are divided into fractions and then these fractions are combined in corresponding share. But this way is difficult and too expensive. In practice specially prepared sand and coarse aggregates with different grading or natural, non-fractionated aggregates are usually used. Therefore, the task is to determine the proportion of each component in an aggregate mix containing N ingredient to provide the best aggregate packing. The following methods of determination optimum aggregate mix design can be mentioned: graphical methods (for example, method of Fuller’s curve approximation carried out by Dutch Shockbeton”[6]), analytical implication of graphical methods [7], analytical calculation by means of least squares [3,7], methods of mathematical iteration, practical and analytical methods based on determination of the maximum packing degree in aggregate mix.

1.2 Determination of optimum aggregate mix by analytical and numerical methods

Conditions of the task: N types of aggregates are given (grading curves of aggregate are determined). Proportion of each aggregate in the mix is to be found to provide the best correlation with an “ideal” curve.

The equation for the real combined grading curve Y_i is as follows:

$$Y_i = \sum_{j=1}^{N} K_j Y_{ji}$$

here K_j - proportion of j-st aggregate in mix;

Yji - real grading of j-st aggregate.

Coefficients K_j can be calculated by minimising the squared sum of deviations between an “ideal” (theoretical) and a real grading curve:

$$\sum_{i=1}^{M} (YT_i - Y_i)^2 \rightarrow \min$$

here M - number of sieves

After differentiation the following system of equations is obtained (in matrix form):

$$\begin{vmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,N-1} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N-1,1} & A_{N-1,2} & \cdots & A_{N-1,N-1} \end{vmatrix} \text{x} \begin{vmatrix} K_1 \\ K_2 \\ \cdots \\ K_{N-1} \end{vmatrix} = \begin{vmatrix} B_1 \\ B_2 \\ \cdots \\ B_{N-1} \end{vmatrix}$$

(matrix of coefficients with unknown members) *(matrix of unknown members)* *(matrix of free coefficients)*

here A and B are the coefficients determined from grading of aggregates.

In the above method the real grading curve is approximated with a preliminarily defined “ideal” curve (all points of curve are preliminarily known). For example, Fuller’s curve is defined with the starting point (X_0, 0) and finishing point (X_{max}, 100). It means, that Fuller’s curve is artificially assigned with the point (X_{max}, 100). But in reality the best correlation between a real and an “ideal” curve may be achieved if X_{max} has another value. This assumption adds just one unknown member to the system of equations (X_{max}) and consequently, adds one equation to the system:

$$\begin{vmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,N-1} & A_{1,N} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,N-1} & A_{2,N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ A_{N-1,1} & A_{N-1,2} & \cdots & A_{N-1,N-1} & A_{N-1,N} \\ A_{N,1} & A_{N,2} & \cdots & A_{N,N-1} & A_{N,N} \end{vmatrix} \text{x} \begin{vmatrix} K_1 \\ K_2 \\ \cdots \\ K_{N-1} \\ T \end{vmatrix} = \begin{vmatrix} B_1 \\ B_2 \\ \cdots \\ B_{N-1} \\ B_N \end{vmatrix}$$

Determination of coefficients and solving of the equation system are being completed by the aid of a computer. This way is quite simple and very quickly gives precise values of coefficients which correspond to an optimum aggregate mix.

But it must be stressed, that very often application of analytical method give undefinite results, for example, proportion of aggregate K_j may be negative or larger than one. Analytical method does not allow to take into consideration the border restrictions on the value of coefficient K_j:

$$0 \leq K_j \leq 1$$

The most stable results are obtained if numerical method of determining an optimum aggregate mix is used. This method provides for calculation the of possible compositions (proportion of each aggregate in mix). As a criterion of an optimum mix is the average squared diviation between real and "ideal" aggregate curves calculated for all sieves:

$$S = \sqrt{\frac{\sum_{i=1}^{M}(YT_i - Y_i)^2}{M - 1}}$$

With the aid of a computer a program for 5 aggregate combinations has been worked out.

In practice from 2 to 4 aggregates are used in a concrete mix. The average squared diviation S is used as a criterion of suitability of the given aggregate combination and allows to compare the possibilities to use different aggregate combinations (Fig. 2).

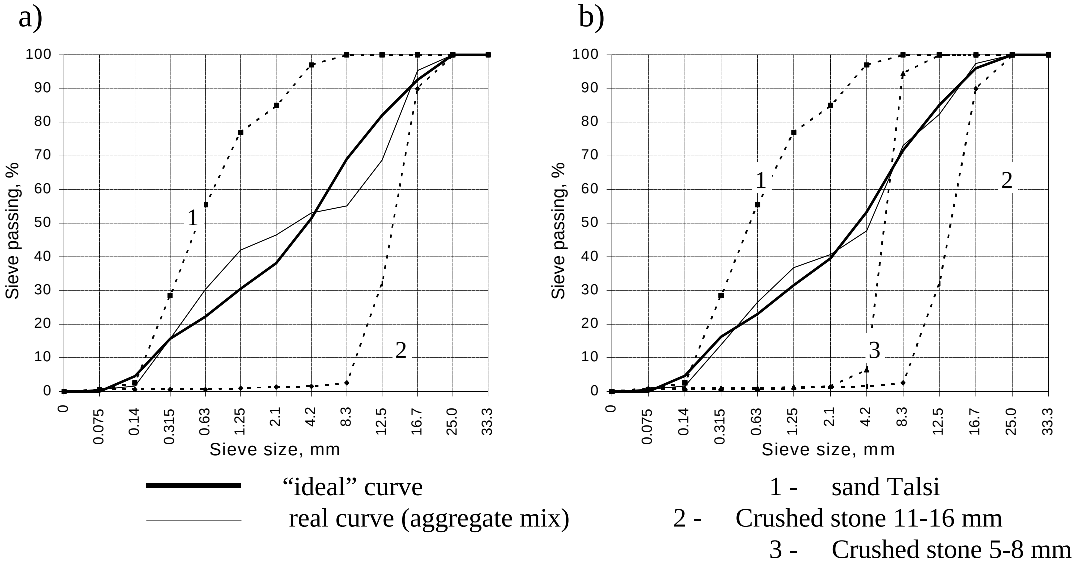

Fig. 2 "Ideal" and real aggregate grading curves

a) *2 aggregates are used (1 type of sand and 1 type of crushed stone), S=8.1%;*

b) *3 aggregates are used (1 type of sand and 2 type of crushed stone), S=3.2%.*

2. OPTIMIZATION OF CONCRETE MIX

2.1 Creation of a Data Base for the existing natural aggregates and "ideal" grading curves

The task of concrete mix optimization is to estimate different concrete compositios with different combination of aggregate and then to choose the best variants of mix by comparing their economical and mechanical properties as well as durability of the material.

The first step in the program of concrete mix optimization is creation of a Data Base for all available aggregates. The Data Base of aggregates includes the following information: name and identification of aggregate, price, parameters of granulation (finess module, number of granulation and coefficient of granulation), grading, density, water absorption, strength and frost resistance. All the data are generalised in Excel worksheets. The first sheet contains sand and gravel-sand mix properties, the second sheet contains the properties of coarse aggregate but the third and the fourth sheets are devoted to "ideal" grading curves and areas.

2.2. Method of concrete mix design

Determination of concrete mix composition includes the following steps:

- Calculation of aggregate mix with the numerical method regarded above;
- Determination of Water-Cement ratio with the aid of widely-used formulas (depending on strength of concrete, strength of cement and quality of aggregates);
- Determination of water consumption, taking into account granulation parameters of aggregates: grading, shape, condition of surface. The main idea of this approach is determination of water absorption capacity (number of granulation) of aggregate taking into account thickness of water film and particle size distribution (grading of aggregate). Number of granulation of the given aggregate is calculated with formula [3]:

$$\lambda_j = \sum_{i=1}^{M} Q_i \lambda_i$$

here Q_i - is the fraction in the sieve i;

λ_i - is the granulation number of the given fraction.

By analogy the granulation number of aggregate mix is determined:

$$\lambda = \sum_{j=1}^{N} K_j \lambda_j$$

Water consumption is determined by formula:

$$W = k \lambda$$

here k - is coefficient determined by consistency of concrete mix.

It must be mentioned that the standard method consider only general characteristics of aggregate – maximum size, finess module, which do not describe the properties of aggregates completely. The granulation method allows to predict properties of concrete more precisely.

2.3 Multipurpose optimization of concrete mixes

2.3.1. Inputting of optimization data:

- properties of concrete and requirements for raw materials: cone slump, compressive strength, type of aggregate, entrained air content, maximum diameter of aggregate.
- available aggregates may be used in possible combinations: numbers of aggregate from Data Base (N1-number of sand; N2-number of course aggregate).

The total number of aggregate mix combinations in case if one type of sand and one type of course aggregate are used (the number of variable $n_x = 2$):

$$n_c = N1 \times N2$$

2.3.2 Concrete mix design for all of aggregate combinations and determination of properties to be optimized:

- Aggregate mix design with numerical method; determination of *criteria of aggregate packing quality*:
  - Average square diviation between real and theoretical grading curve S,
  - Packing degree of aggregate mix (also may be taken into account).
- Concrete mix design; determination of concrete composition in 1m^3. The following properties of concrete quality are to be obtained:
  - Cement consumption in 1m^3 of concrete CEM;
  - Water consumption in 1m^3 of concrete W;
  - Calculated density of concrete D
    This factor must be taken into account if special requirements are specified (for light weight concrete or special heavy concrete).
- Determination of criteria of economy:
  - Cost of raw materials calculated in 1m^3 of concrete COST.
- Determination of the individual purpose function Φ_i for given properties to be optimised by calculating them into relative values from 0 to 1.

2.3.3 Calculation of general purpose function for all combination of aggregate

$$\Phi = \left(\sum_{i=1}^{n_y} \eta_i \, \Phi_i^{-2}\right)^{-1}$$

here n_y - is the number of properties are to be optimized;

η_i - is the preliminary estimated coefficient of significance of given property is to

be optimized ($\sum \eta_i = 1$);

Φ_i - is the individual purpose function for given properties to be optimized.

Comparing purpose functions for different aggregate combinations, the best concrete compositions with biggest Φ value may be determined.
The example of concrete mix optimization are shown in Fig. 3.

3. CONCLUSIONS

Use of numerical method of aggregate mix design with aid of transformed Fuller's curve allows to calculate aggregate mixes for different types of concrete as well as to use natural, non-fractionated aggregates.
Average squared deflection between the "ideal" and the real grading curve S is efficiently used as criterion of packing quality of the aggregate.

Use of granulation number method of concrete mix design allows to predict physical and mechanical properties of concrete (strength and workability) with coefficient of correlation not less then 0.95. At the same time, correlation coefficient between practical and experimental results of the standard method is 0.85 ... 0.9.

System of concrete mix optimization gives a possibility to estimate more objectively and find a compromise variant between economy on the one hand and property on the other hand.

Computer application for concrete mix design allows simply and quickly to choose the optimum variants of physical-mechanical as well as economical properties of concrete.

4. LIST OF NOTATIONS

Y_i	is the sieve passing through of aggregate refering to grading curve
YT_i	is the sieve passing through of aggregate refering to an “ideal” grading curve
K_j	is the proportion of aggregate in mix
S	is the average squared diviation between an “ideal” and a theoretical curve
COST	is the cost of raw materials per $1m^3$ of concrete
CEM	is the cement consumption per $1m^3$ of concrete
W	is the water consumption per $1m^3$ of concrete
N	is the number of aggregate in mix
Φ	is the general purpose function
λ	is the number of granulation of the aggregate mix
k	is the coefficient of consistency of concrete mix

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CONCRETE MIX OPTIMIZATION

Date: 20.05.98.
Time: 10-05

Strength of concrete: 30 MPa
Strength of cement: 40 MPa
Cone Slump: 12 cm
Air: 1 %

Fine aggregate	Course aggregate
4	102
5	107
11	128
12	
29	

The number: 5 (N1) 3 (N2)

n_c = 15
(the number of comb.)

Number of comb.	Fine aggr. No	Course aggr. No	"Ideal" curve S(K1), %	K_1	Row mat. COST, Ls	Cement CEM, kg/m^3	Water W, l/m^3	Density D, kg/m^3	Coefficients of significance					General purpose function
									0.4	**0.4**	**0.1**	**0.1**	**0.0**	
									Individual purpose functions (0 1)					
									S	COST	CEM	U	D	
min:			**4.1**		**17.92**	**305**	**163**	**2349**						
max:			**12.0**		**22.22**	**375**	**210**	**2406**						
1	4	102	9.2	0.420	20.07	336	177	2375	0.36	0.50	0.55	0.70	0.45	0.189
2	4	107	7.0	0.385	18.60	349	184	2354	0.63	0.84	0.37	0.56	0.07	0.379
3	4	128	9.7	0.435	21.08	356	195	2368	0.29	0.26	0.27	0.33	0.33	0.078
4	5	102	7.4	0.465	19.97	333	177	2375	0.58	0.52	0.60	0.71	0.44	0.319
5	5	107	5.4	0.435	18.64	346	184	2354	0.84	0.83	0.42	0.57	0.08	0.492
6	5	128	7.8	0.480	20.90	351	193	2368	0.54	0.31	0.35	0.37	0.33	0.138
7	11	102	6.2	0.510	20.44	305	163	2406	0.74	0.41	1.00	1.00	1.00	0.306
8	11	107	4.1	0.475	19.10	318	171	2385	1.00	0.73	0.81	0.85	0.62	0.690
9	11	128	6.2	0.520	21.31	321	178	2401	0.73	0.21	0.77	0.68	0.91	0.101
10	12	102	8.4	0.520	19.12	337	180	2376	0.45	0.72	0.54	0.64	0.47	0.299
11	12	107	7.0	0.480	17.92	347	185	2358	0.63	1.00	0.40	0.53	0.16	0.422
12	12	128	8.9	0.535	19.93	353	195	2370	0.39	0.53	0.31	0.33	0.37	0.165
13	29	102	11.2	0.375	21.16	355	192	2372	0.09	0.25	0.28	0.39	0.40	0.017
14	29	107	9.1	0.345	19.55	368	198	2349	0.36	0.62	0.10	0.26	0.00	0.064
15	29	128	12.0	0.385	22.22	375	210	2366	0.00	0.00	0.00	0.00	0.29	0.000

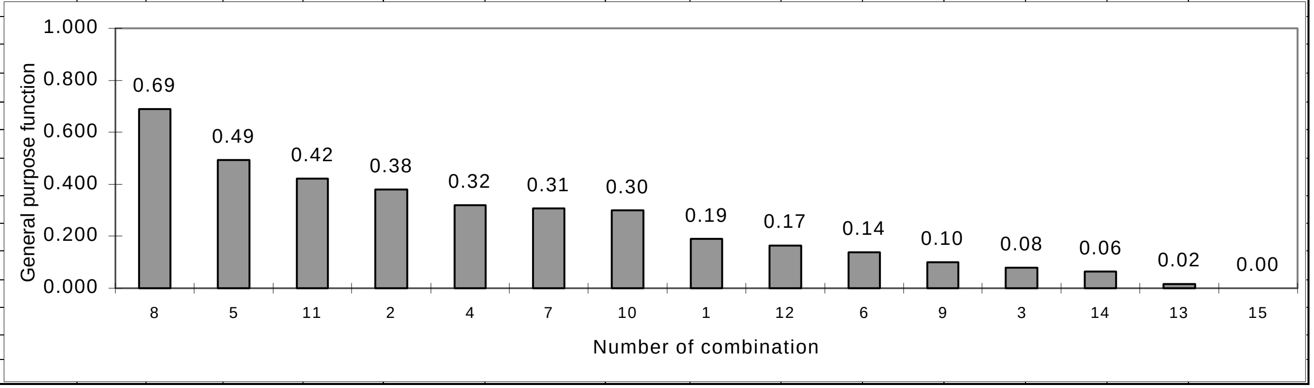

Fig. 3 Example of concrete mix optimization