CONCRETE TIE MODEL FOR THE FLEXURAL BEHAVIOR OF RC AND PC SECTIONS

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SUMMARY

The enhanced concrete tie model described in this paper can be used to determine the flexural behavior of RC and PC sections. This model is applicable to any concrete section, without shape restriction and will be implemented in a shell FE package. To evaluate the performance of the proposed model, a simple program has been developed, using a discretization of the cross section in slices.

Keywords: serviceability behavior, numerical modeling, nonlinear response, cracking, creep, finite element calculation, RC & PC tie

1. INTRODUCTION

Over the past thirty years, several post-tensioned bridges have exhibited an unsatisfactory long-term behavior under service loads, characterized by a non-stabilization of deformation and increased cracking over time. While explanations are available (Favre and al, 1995), some effects cannot be easily taken into account using beam analysis only. It is expected that the finer degree of modeling offered by shell analysis will lead to a better understanding of the actual behavior.

The goal of the research project is to develop a computation tool suitable for use by civil engineers to evaluate the effect of cracking and long-term effects (creep & shrinkage) in shell-type structure, particularly of RC and PC bridges. This will be achieved by implementing a new material model into an existing commercial shell FE program.

The moment-curvature relationship described in the Model Code 90 (Model Code CEB-FIP, 1990) is a well-established tool for the computation of long-term deflections of concrete structures. FE programs using this relationship have already been developed and successfully used for the computation of beam-type structures (Bouberguig and al, 1997). Its validity is however limited to beam-type structures. This research proposes an enhanced concrete tie model, derived from a CEB contribution (CEB 235, 1997). The proposed model accurately describes the behavior of a concrete tie in tension or compression, as well as the behavior of an area of concrete in a section subjected to

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bending with and without axial force. This allows its use to describe the in-plane properties of shell elements, as they appear, for example, in the web or flange of a box-girder section. This model has been extensively compared to the results given by the CEB MC relationship and has given excellent results. These results, as well as the method, are described in this paper.

2. PHYSICAL MODELS

2.1. CEB-FIP Moment-curvature relationship

As described in the Model Code 90, this relationship gives the mean curvature of an RC beam subjected to bending moment with or without an axial compression force. It gives accurate results for compact sections with a vertical symmetry axis. However, it does not include shear effects, nor axial tension.

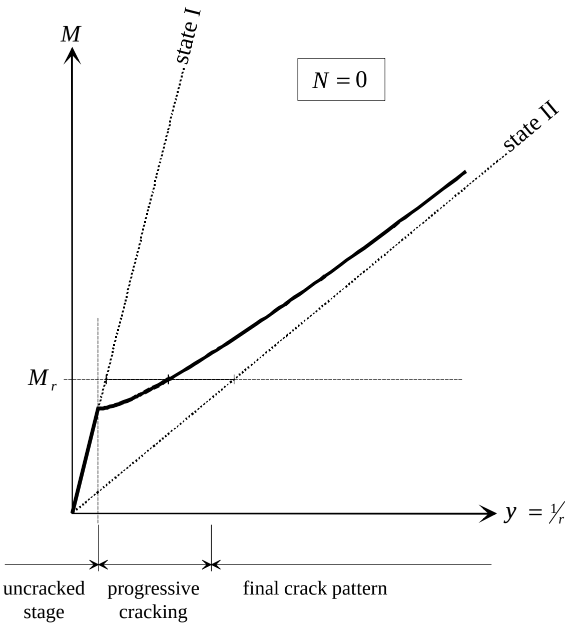


Fig. 1 CEB-FIP Moment-curvature relationship

$$y_m = y_2 - b \cdot (y_{2r} - y_{1r}) \cdot \frac{M_r}{M} \geq y_1$$

M, N = bending moment and normal force applied to the section

y_m = mean curvature under *M, N*

M_r = cracking moment of the section under *N*

y_1, y_2 = state I (uncracked concrete) and state II (fully cracked concrete) curvatures under *M, N*

y_{1r}, y_{2r} = state I,II curvatures under M_r, *N*

b = uncracked concrete contribution coefficient
= 0.8 for first loading
= 0.5 for long term loading

2.2. Concrete tie model

The enhanced concrete tie model is derived from the relationship proposed by Sippel (CEB 235, 1997). Three modifications are introduced in the relationship itself.

1. In the original model, the stabilized-crack branch of the relationship is parallel to the state II. Since the moment-curvature relationship is asymptotical to the state II, the stabilized-crack branch has been modified as shown in fig. 2, as proposed by Bruggeling (Bruggeling, 1991) for prestressed beams.

2. Some areas of concrete within a section contain little or no reinforcement. In concrete areas with a low reinforcement ratio, the tensile strength after the first crack occurs is taken equal to the yielding force of the reinforcement (fig. 5, curve c). If this value is very low or zero, it is taken as a fraction of the tensile strength (fig. 3). This applies especially to the intermediate zones of the section where there is no particular reinforcement but the strains on the section are controlled by the main reinforcement near tension face.

3. For long term loading, Sippel (CEB 235, 1997) gives a logarithmic formulation for the reduction of the tensile strength of concrete. For practical calculation with the proposed model, values of k between 0.6 and 0.8 have given good results.

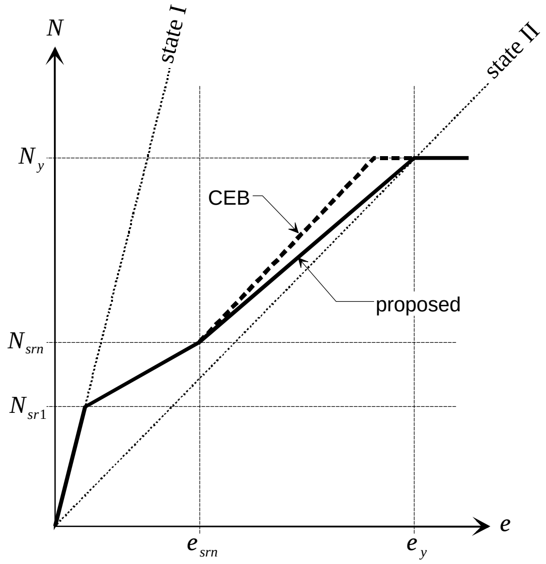


Fig. 2 Modification of the final crack pattern stage

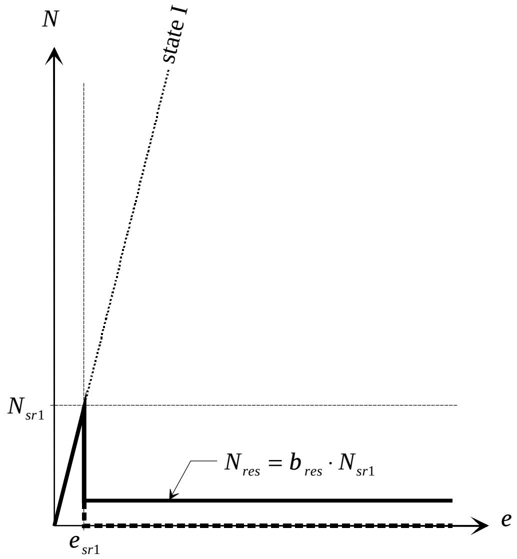


Fig. 3 Residual tensile strength of cracked non reinforced slice

Hence the following formulation, for a reinforced concrete tie at service state (fig. 4):

Uncracked Stage, including compression:

$$N = EA_I \cdot e \quad \text{if} \quad e < e_{sr1}$$

with $e_{sr1} = k \cdot f_{ct,fl\,5\%} / E_c$

k = reduction factor for the tensile strength under long term loading
= 1.0 for short term loading
= 0.3 to 0.5 for long term loading

Progressive cracking:

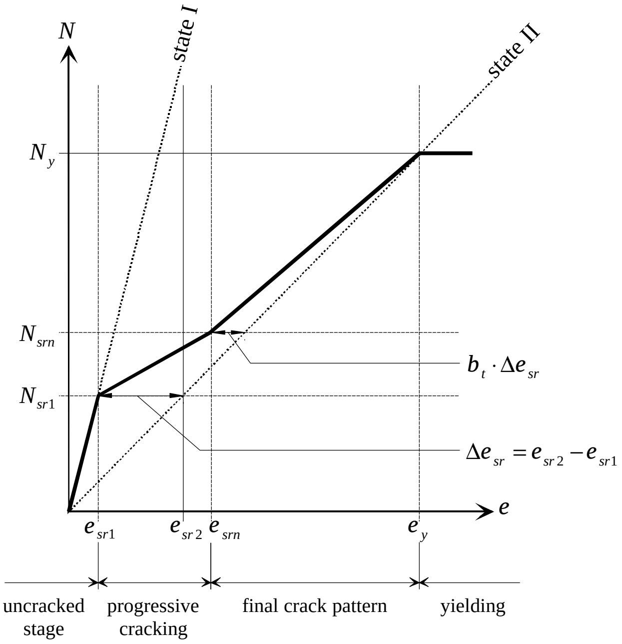
$$N = N_{sr1} + \frac{N_{srn} - N_{sr1}}{e_{srn} - e_{sr1}} \cdot (e - e_{sr1}) \quad \text{if} \quad e_{sr1} < e < e_{srn}$$

with $N_{sr1} = e_{sr1} \cdot EA_I$ (first crack)

$N_{srn} = N_{sr1} \cdot f_{ct,fl\,95\%} / f_{ct,fl\,5\%}$ (last crack)

$$e_{sr2} = N_{sr1} / EA_{II}$$
$$e_{srn} = N_{srn} / EA_{II} - b_t \cdot (e_{sr2} - e_{sr1})$$
$b_t = 0.40$ first loading

$b_t = 0.25$ long term loading or cyclic loading



$f_{ct,fl}$ = mean flexural tensile strength of concrete

$f_{ct,fl5\%} = 0.75 \cdot f_{ct,fl}$

$f_{ct,fl95\%} = 1.25 \cdot f_{ct,fl}$

A_c = concrete area

A_s = reinforcement area

E_c = $E_c(t)$ E-modulus of concrete

E_s = E-modulus of steel

EA_I = $E_c \cdot (A_c - A_s) + E_s \cdot A_s$

EA_{II} = $E_s \cdot A_s$

N = force in the RC slice

e = strain in the RC slice

Fig. 4 Modified relationship for a typical reinforced concrete section

Final crack pattern:

$$N = N_{srn} + \frac{N_y - N_{srn}}{e_y - e_{srn}} \cdot (e - e_{srn}) \quad \text{if} \quad e_{srn} < e < e_y$$

with $e_y = f_y / E_s$

$N_y = f_y \cdot A_s$

Yielding of reinforcement:

$$N = N_y \quad \text{if} \quad e_y < e$$

Check for yielding of reinforcement (low reinforcement ratio, fig. 5**):**

If $e > e_{sr1}$ and $N_y < N_{sr1}$ then $\underline{N \le N_y}$

Check for residual tensile force of the slice (fig. 3)**:**

If $e > e_{sr1}$ then $\underline{N \ge N_{res} = b_{res} \cdot N_{sr1}}$

The constant b_{res} represents the contribution of uncracked concrete in zones with little or no reinforcement. A reasonable value was obtain through calibration of the numerical model:

$$b_{res} = 0.12$$

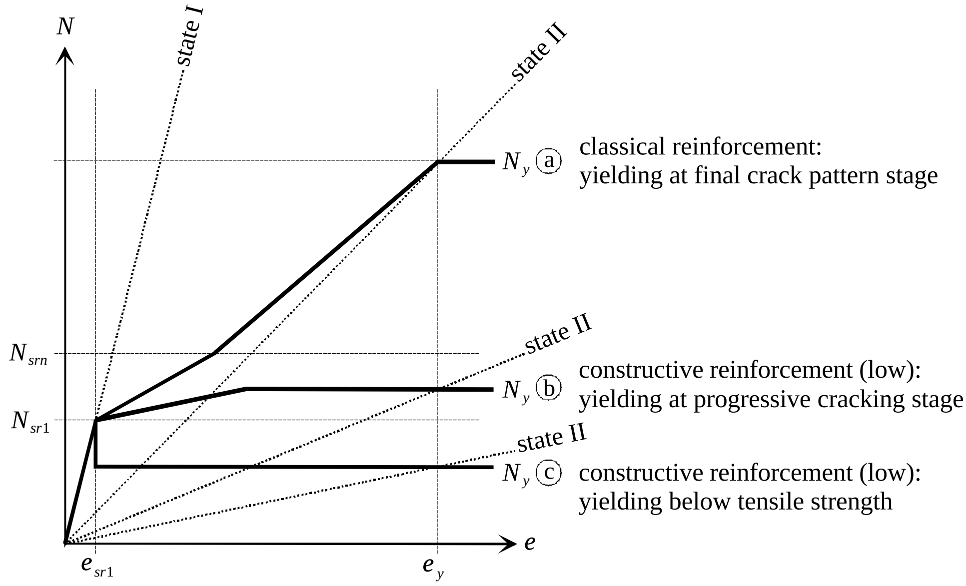
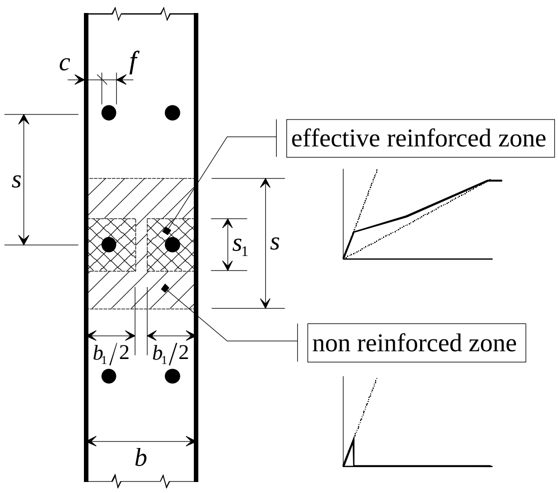


Fig. 5 Force-strain relationship for several reinforcement ratios of the same concrete section

Spacing of the reinforcement bars

To account for the effect of spacing of reinforcement bars, the force-strain relationship is not applied to the whole concrete area, but only to an effective reinforced zone, as shown in fig. 6. The rest of the concrete is taken in account only when it is uncracked ($\varepsilon < \varepsilon_{sr1}$).



b = thickness
b_1 = effective thickness
s = spacing of bars
s_1 = effective height of the spacing
c = cover
f = diameter of bars
n = number of reinforcement layers - usually 2

$$s_1 = 2.5 \cdot \left(c + \tfrac{1}{2} f\right) \leq s \qquad \text{(Model Code 90)}$$

$$b_1 = n \cdot 2.5 \cdot \left(c + \tfrac{1}{2} f\right) \leq b \qquad \text{(by analogy)}$$

Fig. 6 Effective application zone of the concrete tie force-strain relationship

3. APPLICATION

The AlphaFlex package is a Fortran-90 code developed at IBAP as a benchmark for testing models with RC sections subjected to a bending moment and an axial force. The resolution algorithm allows the use of any non-linear material model. The main assumptions are a vertical axis of symmetry of the section and the conservation of plane sections. This package was specifically designed for large parametric studies.

The enhanced force-strain relationship described above has been implemented in this package, as well as the CEB M-C relationship for comparison. A parametric study was performed on a family of 1100 rectangular sections, and another family of 2900 T-sections, including variations of the following parameters:

- Section geometry
- Reinforcement (area, diameter, spacing, cover)
- Tensile strength of concrete
- Amount of prestressing (from 0 to 5 N/mm^2)
- Concrete tie model parameters (b_t , k , b_{res})

3.1. Reconstructed moment-curvature relationship

For each section the complete moment curvature relationship was computed using both CEB M-C relationship and numeric integration across the section of force-strain relationship. Figures 7 and 8 show these results for a rectangular RC section and a PC T-section respectively.

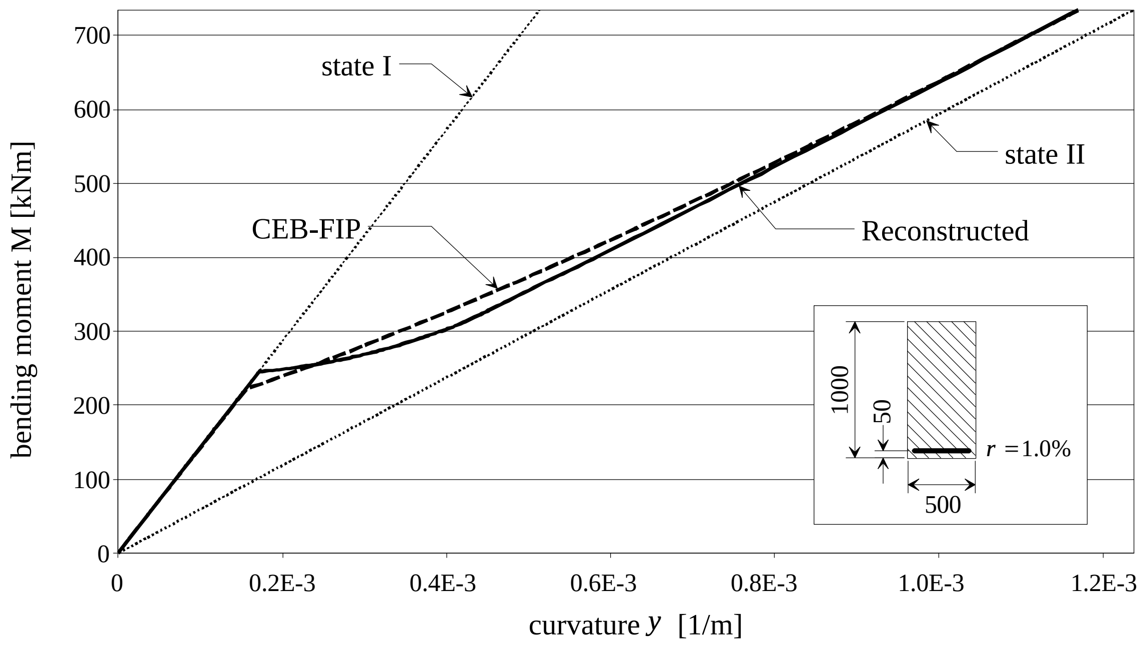


Fig. 7 Moment-curvature relationships for a rectangular RC section

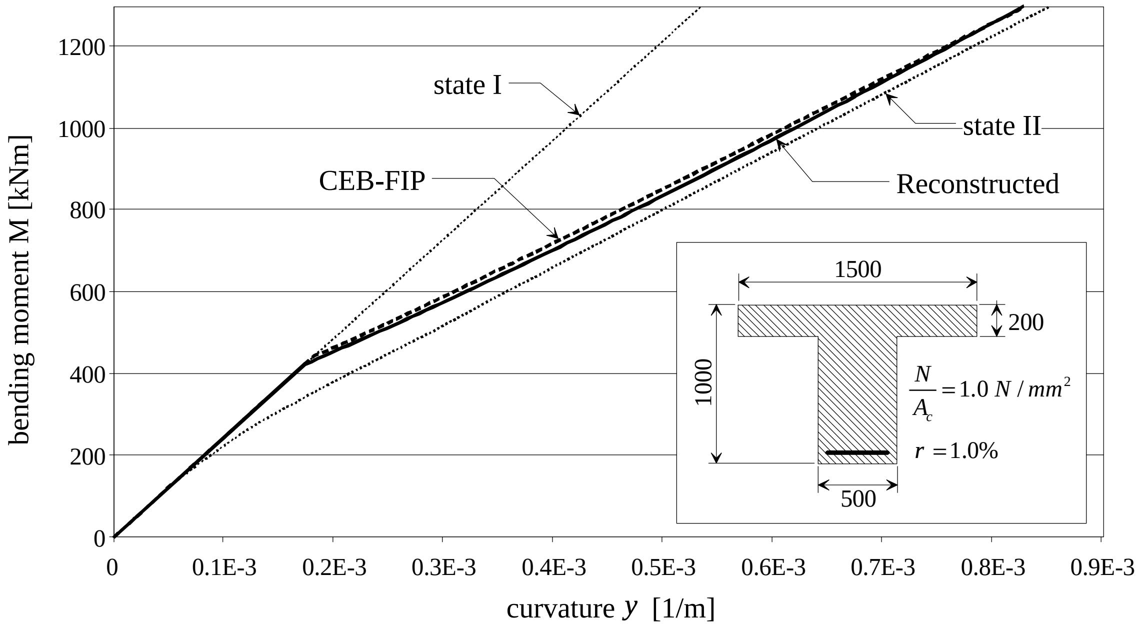


Fig. 8 Moment-curvature relationships for a PC T-section

3.2. Global results

For comparison, the deflection of a simply supported beam subjected to twice its cracking load was computed using both CEB moment-curvature relationship and enhanced concrete tie model. A deviation ratio has been defined for each section as the relative difference between these two results.

All sections gave a deviation lower than 25% (14% for rectangular sections) and more than 80% of these sections gave less than 5% deviation.

3.3. Non prestressed sections

The amount of prestressing appears to be the most important parameter. Nevertheless other parameters are significant for non-prestressed RC sections.

The reinforcement ratio is the second most important parameter. The sections with higher reinforcement ratio showed an excellent accuracy while the section with lower ratio appeared to be too stiff when compared to the CEB M-C relationship.

In third, the geometrical characteristics of the sections have also a slight influence. The accuracy was better for sections with thin web or flange. The reduction of the size of the cracked zone naturally leads to a better agreement of both models, since the section is then closer to state I, which gives exactly the same results with both models.

3.4. Effect of prestressing

As a general comment, an excellent agreement was observed for prestressed sections. This is mainly due to the fact, that these sections are less cracked than RC sections. The phenomenon explained above for sections with thin web or flange is here of great importance.

Another reason is the non-linearity of the state II relationship induced by the presence of an axial force. There is no hard discontinuity when the curve leaves the state I branch, which is numerically favorable. For non-prestressed sections, both CEB and reconstructed M-C relationship show a sharp angle at this point. The deviation, when there is any, is then strongly softened.

4. CONCLUSIONS

The proposed enhanced concrete tie model has shown excellent results. The best fit was obtained for prestressed sections and for non-prestressed sections with higher reinforcement ratio and with thin web or flange. This model is thus particularly suitable for computation of structures such as bridges with a box-girder section using a shell modelization.

The concrete tie model has now been implemented in the finite element shell program. A few results have already been obtained and compared to available experimental results, showing good concordance with no particular calibration of the model.

In the coming months, the program will be applied to special cases of long-term behavior. In particular, the effect of shrinkage-induced cracking of deck slabs will be investigated.

5. REFERENCES

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