COMPARISON OF THE RESULTS OF NOTCHED THREE POINT BENDING TEST WITH MODEL CODE 2010 FORMULAS

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The primary application of the notched three point bending test (3PBT) is to determine the fracture energy of concrete. However, the measurement setup is also suitable for determining additional mechanical parameters: flexural tensile strength, modulus of elasticity, and indirectly the compressive strength also. The aim of this paper is to present the calculation methods of the mechanical properties that can be determined from the results of a test series in which mixtures with different types of aggregates were used (quartz, dolomite, limestone, andesite, expanded clay). To validate the obtained results, the parameters determined from the measurements are compared to the formulas of the fib Model Code 2010. A recommendation is also presented for the calculation of the fracture energy by using compressive strength values measured on a half prism.

Keywords: thermally damaged concrete, three point bending test (3PBT), crack mouth opening displacement (CMOD), fracture energy

1. INTRODUCTION

Depending on the shape and the behaviour of the fracture process zone around a crack tip, construction materials can be classified as brittle, quasi-brittle and elastic-plastic. Concrete belongs to the quasi-brittle category (*Khalilpour, BaniAsad, and Dehestani*, *2019*; *Rao and Rao*, *2014*). Concrete, as a construction material, contains micro-cracks, pores, potential failure locations even without loads (*Sólyom, Di Benedetti, and Balázs*, *2021*). During loading the number of cracks and failure locations increase and affect the behaviour and load-bearing capacity of the material (*Fehérvári, Gálos, and Nehme*, *2010a*). As a result of the internal forces, the micro-cracks start to form larger cracks and above a critical level crack opening and propagation accelerate (*Bažant and Planas*, *1997*). Understanding the behaviour of cracks is essential for the applicability of structural materials, as failure processes begin at potential failure locations (*Griffits*, *1921*). Fracture mechanics deals with the analysis of stress conditions around cracks and with the determination of the parameters affecting the opening and propagation of the cracks. In fracture mechanics the toughness of materials is most frequently characterised by two parameters: fracture energy (G_f) and critical stress intensity factor (K). Fracture energy is the energy required for the opening and propagation of the unit area of a crack (*Hillerborg, Modéer, and Petersson*, *1976*; *Khalilpour et al.*, *2019*), while the critical stress intensity factor characterises the resistance against rapid, uncontrolled crack propagation, introduced by *Irwin* (*Irwin*, *1957*), who also classified cracks by the main failure mode causing them: mode I is opening (tension), mode II is sliding (in plane shear), mode III is tearing (out of plane shear).

In case of concrete and reinforced concrete structures, typically tensile cracks (mode I) are analysed. The most common experimental method to investigate this failure is the notched 3PBT (*Khalilpour et al.*, *2019*). Prior to the test a crack-starting notch is made in the concrete specimen (it can be formed by sawing before testing or already during concreting), the height of the notch depends on the applied standards and recommendations, typically 1/6 - 1/2 of the specimen's height is used (*EN 14651:2005+A1*, *2007*; *Hillerborg*, *1985*; *JCI-S-001-2003*, *2003*; *RILEM Technical Committee 50 FMC*, *1985*). There are different versions of the experimental setup of 3PBTs. If the supports are located close to the edges of the specimen, then after the opening of a certain critical crack (*Fig. 1a*), the specimen cracks due to gravity, independently of all other loads. In this case the total fracture energy cannot be measured, but it can be corrected during the evaluation of the results (*JCI-S-001-2003*, *2003*; *RILEM Technical Committee 50 FMC*, *1985*). In order to balance the effect of self-weight, the measurement setup can also be designed so that the specimen extends significantly beyond the supports (*Fig. 1b*) or additional weights are placed at the ends of the beam (*Fig. 1c and 1d*) (*Kaplan*, *1961*). The adequate size of the specimen highly depends on the maximum aggregate size (d_{max}). Typically, even the smallest dimension of the specimen should be larger than 4 times d_{max} (*JCI-S-001-2003*, *2003*); otherwise, the aggregate size will affect the value of the fracture energy.

The primary application of the notched 3PBT is to determine the fracture energy of concrete. However, the measurement setup is also suitable for determining additional mechanical parameters: flexural tensile strength, modulus of elasticity, and indirectly also the compressive strength. The aim of this paper is to present the calculation methods of the mechanical properties that can be determined from the test results of 3PBTs.

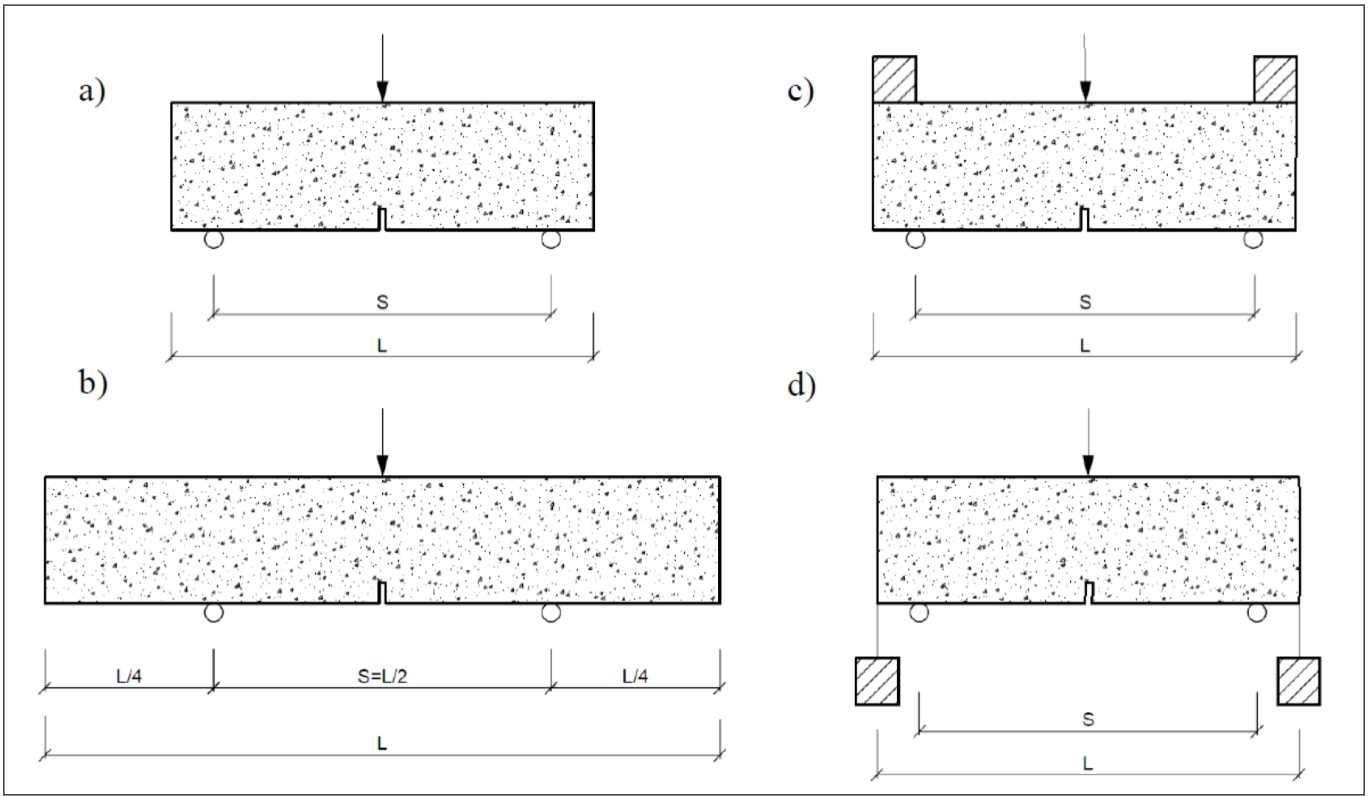


Fig. 1: Notched 3PBTs: a) general; take into account the gravity b) with overhang, c) and d) with weights

2. EXPERIMENTAL DETAILS

2.1 Materials

During our tests, 7 types of aggregates were used. In normal concretes (NCs), the most typical river quartz gravel, sand and mined dolomite, limestone and andesite were used, while in lightweight concretes (LWCs), two types of expanded clay aggregates (ECAs) were applied.

The shape of quartz aggregate and ECA was rounded: in case of quartz due to river fragmentation, while in case of ECA due to the technological process. The shape of the dolomite, limestone and andesite aggregate was angular due to the crushing process.

In case of the limestone aggregate, the body density was 2710 kg/m^3, which was available from the data given by the mine. For the other aggregates, the body density was determined by own measurements. The body density of the quartz gravel aggregate was 2645 kg/m^3, that of the dolomite aggregate was 2850 kg/m^3, and that of the andesite aggregate was 2700 kg/m^3. The body density of ECAs was 1465 kg/m^3 in case of type D1 and 1048 kg/m^3 in case of type D2. In case of lightweight aggregates, their high porosity causes high water absorption, which also affects the water-cement ratio of the concrete mixture *(Nemes and Józsa, 2006)*; therefore, the ECAs were saturated with water. After 30 minutes of water absorption, the body density of type D1 was 1549 kg/m^3, while that of type D2 was 1262 kg/m^3. The amount of absorbed water was taken into account during the correction of the concrete mixtures.

The type of cement applied in our mixtures was CEM III/32.5 R containing slag. The water-cement ratio was 0.45. The consistency of fresh concretes was F4 (*EN 12350-5:2019*, *2019*) which was regulated by the addition of superplasticiser (BASF Glenium C300). The fraction 0/4 mm was quartz sand in all the mixtures when fraction 4/8 mm was used. In order to make the comparison of the mixtures with different aggregates possible, it was important to evolve a similar aggregate skeleton; therefore, similar cement content, aggregate content and size distribution (0/4 mm 43%; 4/8 mm 57%) were applied. The concrete composition of different mixtures is summarised in *Table 1.*

The casted specimens were stored under water for 7 days and then in a climate chamber (temperature: 20 °C, relative humidity: 50 %). Tests of specimens were performed at 60 days of age.

2.2 Test equipment

In order to determine the fracture energy, crack mouth opening displacement (CMOD) controlled 3PBTs were carried out. The test setup is shown in *Fig.* 2. The applied CMOD controlled method is different from the previously recommended crosshead displacement controlled method (*Hillerborg*, *1985*; *RILEM Technical Committee 50 FMC*, *1985*), but it is accepted by many standards (*EN 14651:2005+A1*, *2007*; *JCI-S-001-2003*, *2003*). The force-CMOD curves derived by the two methods are the same (*Lee and Lopez*, *2014*). The size of the applied specimens was 70x70x250 mm. Based on literature this size is big enough to make the effect of aggregate size negligeable in case of 8 mm maximum aggregate size (*Fehérvári, Gálos, and Nehme*, *2010b*; *Hillerborg*, *1985*; *JCI-S-001-2003*, *2003*; *RILEM Technical Committee 50 FMC*, *1985*). The distance between the supports was 200 mm. The width of the notch was 4 mm, and its height was one sixth (12.5 mm) of the total height of the specimen. Crosshead displacement, CMOD and force were detected during the tests. During loading, the rate of CMOD was kept constant (0.01 mm/s).

The compressive strength of the concrete mixtures was determined by two methods. The compressive strength of cubes with the size of 150x150x150 mm was measured

Table 1: Concrete mix designs for 1 m³ (quantities are in kg)

Unit weight		Mixture symbol							
		4S	4S8Q	4D	4S8D	4S8L	4S8A	4S8D1	4S8D2
Aggregate	sand (0/4 mm)	1391	782	-	782	782	782	782	782
	quartz (4/8 mm)	-	1037	-	-	-	-	-	-
	dolomite (0/4 mm)	-	-	1499	-	-	-	-	-
	dolomite (4/8 mm)	-	-	-	1118	-	-	-	-
	limestone (4/8 mm)	-	-	-	-	1063	-	-	-
	andesite (4/8 mm)	-	-	-	-	-	1059	-	-
	expanded clay D1 (4/8 mm)	-	-	-	-	-	-	574*	-
	expanded clay D2 (4/8 mm)	-	-	-	-	-	-	-	411*
Cement (CEM III/32.5 R):		600	390	600	390	390	390	390	390
Water:		270	175	270	175	175	175	175	175
w/c ratio		0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
Superplasticizer:		0.3	0.78	0.45	1.79	2.4	3.5	0.4	0.2

** calculated with dry body density of aggregate*

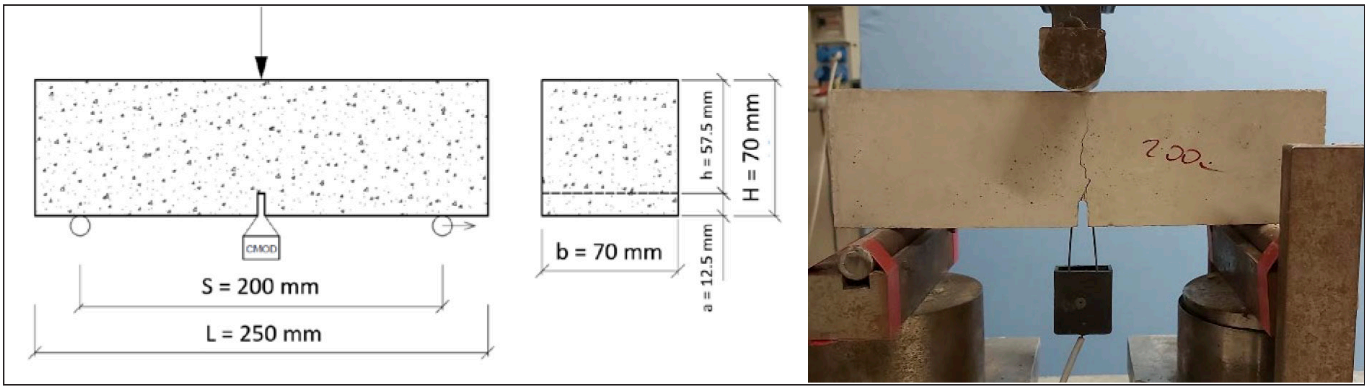


Fig. 2: Setup of the 3PBT

according to the standard *EN 12390-3* (*EN 12390-3:2019*, *2019*). Compressive strength was also measured on half prisms previously subjected to 3PBT (*Fig.* 3) (*Alimrani and Balazs*, *2020*; *Lublóy, Balázs, and Czoboly*, *2013*), where the load was transferred by steel plates with the size of 70x70 mm, which was the same as the width of the prism.

Mechanical tests were extended by measurements of moisture content and apparent porosity, which were measured on one half of the prisms. To determine moisture content, half prisms were dried in a drying furnace at 60 °C until constant mass. Initial moisture content could be calculated by the difference between the original and the dried mass. After drying, the specimens were stored under water until constant mass, therefore the amount of water uptake could be measured, and the volume of open pores could be determined (*EN 1936:2007*, *2007*).

Fig. 3: Compressive strength test: a) cube; b) half prism

3. RESULTS AND DISCUSSION

3.1 Body density, moisture content and apparent porosity

Dry body density values (*Table 2*) of the mixtures with ECA (4S8D1, 4S8D2) were lower than 2000 kg/m^3, therefore they could be considered as LWC (*fib BULLETIN 8*, *2000*; *fib MC2010*, *2013*). As expected, the apparent porosity of LWC mixtures was high due to the high porosity of aggregate particles. The apparent porosity of the mixture containing quartz sand only (4S) and dolomite sand only (4D) was also high. The mixtures with dolomite gravel (4S8D) and andesite gravel (4S8A) had the highest body density and consequently the lowest apparent porosity and moisture content.

3.2 Compressive strength

The *fib* Model Code 2010 (*fib MC2010*, *2013*) calculates the mechanical parameters by using the compressive strength of concrete. Therefore, in addition to 3PBTs, compressive strength tests were also performed using cubes and half prisms previously subjected to 3PBT. The results of the measurements are summarised in *Table 2* and *Fig. 4.*

As expected, the LWCs had the lowest compressive strength. The compressive strength of the mixture with lower body density ECA (4SD2) was significantly lower than that of the mixture with higher body density ECA (4SD1). The mixtures with andesite gravel (4S8A) and with limestone gravel (4S8L) had the highest compressive strength, which is well reflected in the high body density of both mixtures.

The results show that the values measured on cubes and on half prisms were close to each other. Typically, the variance of the values measured on half prisms was larger than that of the values measured on cubes. Based on the value pairs shown, the compressive strength values measured on half prisms were lower than the values measured on cubes if the latter was below 66 N/mm^2. Above 66 N/mm^2 the prisms had higher compressive strength.

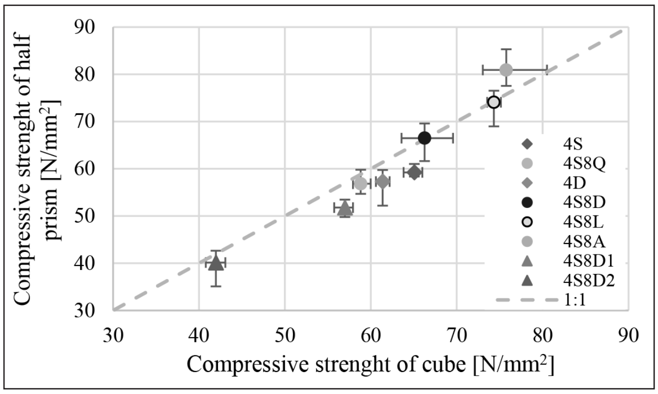


Fig. 4: Compressive strengths of the mixtures

3.3 Flexural tensile strength

From the data measured during the notched 3PBTs the flexural tensile strength of the mixture could also be directly calculated by using the following formula:

$$f_{ct,fl} = \frac{3FS}{2bh^2} \quad \text{(EN 14651:2005+A1, 2007)} \qquad (1)$$

where:
- $f_{ct,fl}$ flexural tensile strength [N/mm^2],
- F the force of rupture [N]
- S loading span (200 mm),
- b width of specimen (70 mm),
- h specimen's height above the notch (57.5 mm).

The flexural tensile strength values calculated from the data measured during the notched 3PBTs are summarised in *Table 2*.

The *fib* Model Code 2010 provides a formula for the calculation of the pure tensile strength of concrete, which can be converted to flexural tensile strength by a factor depending on the width of the specimen (α_{fl}). The formula is as follows, if the concrete strength class is higher than C50/60:

$$f_{ctm,fl} = \frac{2.12*\ln(1+0.1(f_{ck}+\Delta f))}{\alpha_{fl}} \quad \text{(fib MC2010, 2013)} \qquad (2)$$

$$\alpha_{fl} = \frac{0.06b^{0,7}}{1+0.06b^{0,7}} \quad \text{(fib MC2010, 2013)} \qquad (3)$$

where:
- $f_{ctm,fl}$ mean flexural tensile strength [N/mm^2],
- f_{ck} characteristic compressive strength [N/mm^2],
- Δf 8 N/mm^2,
- b width of specimen (70 mm).

It can be seen in the formula (Eq. 2) that 8 N/mm^2 is added to the characteristic value of the compressive strength of the concrete, which will thus correspond to the mean compressive strength.

In the case of LWC, the recommended relation also takes into account the body density of the concrete:

Table 2. Average values of measured non-mechanical and mechanical properties of the mixtures (each value is the average of 3 or 4 measurements)

Mixture	Dry body density [kg/m^3]	Moisture content [%]	Apparent porosity [%]	Compressive strength [N/mm^2]		Flexural tensile strength [N/mm^2]	Modulus of elasticity [N/mm^2]	Fracture energy [N/m]
				cube	half prism			
4S	2090.4	5.67	16.12	65.06	59.23	4.41	20432	116.05
4S8Q	2230.4	3.20	13.16	58.82	56.84	5.82	23661	180.10
4D	2139.1	4.17	18.31	61.37	57.30	4.42	20172	92.81
4S8D	2301.9	2.60	11.27	66.26	66.47	9.02	30950	146.32
4S8L	2302.6	2.39	10.21	74.32	74.08	8.91	35579	121.10
4S8A	2260.6	3.21	11.62	75.77	80.91	8.77	30453	158.12
4S8D1	1764.8	5.05	16.59	56.98	51.78	2.45	10442	80.16
4S8D2	1715.3	5.34	17.23	41.96	40.15	2.21	8460	95.11

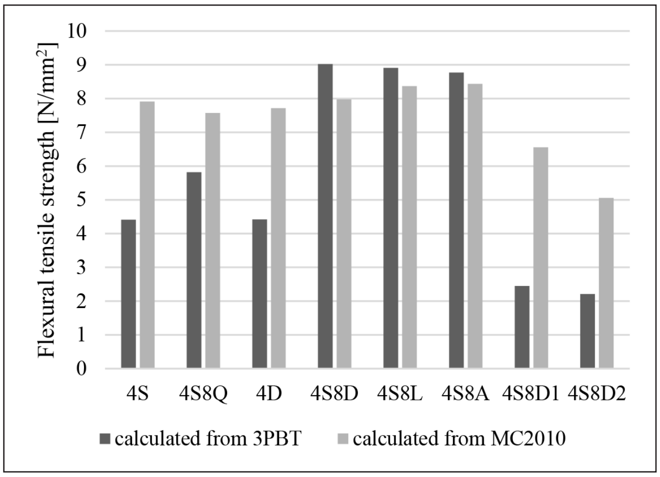


Fig. 5: Flexural tensile strength of the mixtures

$$f_{lctm,fl} = \frac{\mu_l * 0.3(f_{ck})^{2/3}}{\alpha_{fl}} \text{ (fib MC2010, 2013)} \quad (4)$$

$$\mu_l = (0.4 + 0.6\frac{\rho}{2200}) \text{ (fib MC2010, 2013)} \quad (5)$$

where: $f_{lctm,fl}$ mean tensile strength of LWC [N/mm²],
f_{ck} characteristic compressive strength [N/mm²],
α_{fl} from Eq. 3,
ρ oven-dry density of the lightweight aggregate concrete [kg/m³].

The values calculated from the data measured during the notched 3PBTs and the flexural tensile strength values obtained from the previously presented formulae of the Model Code are compared in *Fig. 5.*

The results in *Fig. 5* show that the flexural tensile strength values directly calculated by 3PBT and by the compressive strength were close in case of crushed stone aggregate concretes (4S8D, 4S8L, 4S8A), where the difference was only 3-10%. In these cases, the flexural-tensile strength values directly calculated by 3PBT exceeded the ones calculated by compressive strength, so the formulas of the *fib* Model Code 2010 gave a safe approximation. In other cases, the values directly calculated by 3PBT were overestimated by the formulae of the Model Code: in the case of concrete with quartz gravel aggregate (4S8Q) and the two mixtures with 0/4 mm fractions only (4S, 4D), the difference was 30-80%. In case of LWCs, the difference of values calculated by 3PBT and using the compressive strength was even more significant and could reach 130-170%. It is important to note that the formulas proposed by the *fib* Model Code 2010 do not correspond to bending tests performed on notched specimens. In case of notches, stress concentrations may change the behaviour of the material.

3.4 Modulus of elasticity

From the data measured during the notched 3PBTs, the modulus of elasticity of the mixture could also be directly calculated by using the following formula:

$$E = \frac{6SaV_1(a/H)}{C_i H^2 b} \text{ (Surendra, 1990)} \quad (6)$$

$$V_1\left(\frac{a}{H}\right) = 0.76 - 2.28\left(\frac{a}{H}\right) + 3.87\left(\frac{a}{H}\right)^2 - 2.04\left(\frac{a}{H}\right)^3 + \frac{0.66}{(1-(\frac{a}{H}))^2}$$

(*Tada, Paris, and Irwin*, *2000*) (7)

where: E modulus of elasticity [N/mm²],
S loading span (200 mm),
a height of the notch (12.5 mm),
C_i the initial compliance calculated from the load-CMOD curve [mN⁻¹]
H height of specimen (70 mm),
b width of specimen (70 mm).

The V_1(a/H) is the geometric function, which describes the relationship between the dimensions of the test specimen and the notch. The coefficient C_i in the formula takes into account the initial slope of the force-CMOD curve, which was determined by fitting a line to 40% of the force of rupture. The calculated values of the modulus of elasticity are summarised in *Table 2*.

The *fib* Model Code 2010 also uses the compressive strength of concrete to determine the modulus of elasticity by the following formula:

$$E_c = E_{c0}\alpha_E(\frac{f_{cm}}{10})^{1/3} \text{ (fib MC2010, 2013)} \quad (8)$$

where: E_c mean modulus of elasticity [N/mm²],
E_{c0} 21.5*10³ N/mm²,
f_{cm} mean compressive strength [N/mm²]
α_E reduction factor depending on the aggregate type.

In case of LWC, the previously presented equation (Eq. 8) is modified by a multiplication factor depending on the density of the mixture:

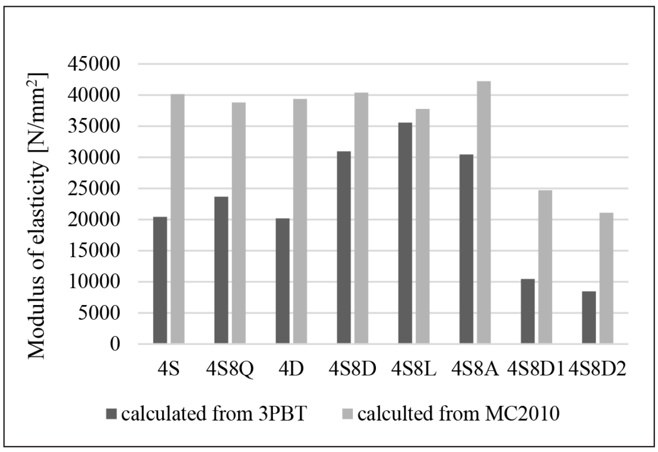
$$E_{lc} = \mu_E E_c \text{ (fib MC2010, 2013)} \quad (9)$$

$$\mu_E = (\frac{\rho}{2200})^2 \text{ (fib MC2010, 2013)} \quad (10)$$

where: E_c calculated from Eq. 8
ρ oven-dry density of the lightweight aggregate concrete [kg/m³].

The modulus of elasticity values calculated by the results of 3PBT and the formulas of the *fib* Model Code 2010 are compared in *Fig. 6.* It can be seen that the best agreement between the results of the two calculation methods occurred in the case of crushed stone aggregate concretes. For the other mixtures, the difference was significant (95-150%). It is important to note that in the case of a notched specimen, stress concentration can occur, which can change the behaviour of the material, even the elastic behaviour of the concrete in the zone around the crack tip. Therefore, in case of lightweight aggregate and small aggregate size (d_{max}=4 mm), during the determination of the modulus of elasticity, it is recommended to treat the results obtained from 3PBT with caution, and rather to determine the modulus of elasticity by a standard test setup.

Fig. 6: Modulus of elasticity



3.5 Fracture energy

During the research, the fracture energy was determined by the following formula

$$G_F = \frac{0.75W_0 + 0.75\left(\frac{S}{L}m\right)g * CMOD_c}{b * h} \quad (JCI\text{-}S\text{-}001\text{-}2003, 2003) \quad (11)$$

where G_F fracture energy [N/m],
W_0 area below the force-CMOD curve up to rupture of the specimen [Nm],
S loading span (200 mm),
L total length of specimen (250 mm),
m mass of specimen [kg],
g gravitational acceleration (9.807 m/s²),
$CMOD_c$ crack mouth opening displacement at the time of rupture [mm],
b width of specimen (70 mm),
h specimen height above the notch (57.5 mm).

The applied formula also takes into account the effect of the gravitational force acting on the specimen. Due to the test setup, the crack opening was affected not only by the loading itself but also by the self-weight of the specimen. Consequently, if the crack opening was in the critical phase, the self-weight itself could cause failure, therefore the total fracture energy could not be measured by the setup. Thus, correction by gravitational force was also required during the calculation. The average of the fracture energy values is summarised in *Table 2*.

The *fib* Model Code 2010 determines the fracture energy for NC from the average compressive strength of the concrete with the following formula:

$$G_F = 73 f_{cm}^{0.18} \quad (fib\ MC2010, 2013) \quad (12)$$

where: G_F fracture energy [N/m],
f_{cm} compressive strength [N/mm²].

In case of LWC, another formula is used, where the average tensile strength of the concrete is included:

$$G_{F,l} = G_{f0A} + 16 f_{lctm} \quad (fib\ MC2010, 2013) \quad (13)$$

where: $G_{F,l}$ fracture energy of LWC [N/m],
$G_{F,0A}$ 24 N/m for LWC with normal weight sand
f_{lctm} tensile strength of LWC [N/mm²].

Previously, the formula of Model Code 1990 (*CEB-FIP, 1993*) also used the mean compressive strength of concrete, but also took into account the maximum aggregate size of the aggregate. However, this relationship corresponded only to NCs:

$$G_F = G_{F0}(f_{cm}/f_{cm0})^{0.7} \quad (CEB\text{-}FIP, 1993) \quad (14)$$

where: G_F fracture energy [N/m],
G_{F0} 0,025 N/mm, in case of 8 mm maximum aggregate size,
f_{cm} compressive strength [N/mm²],
f_{cm0} 10 N/mm².

The fracture energy calculated by the previous formulae (Eqs. 11-14) is summarised in *Fig. 7*.

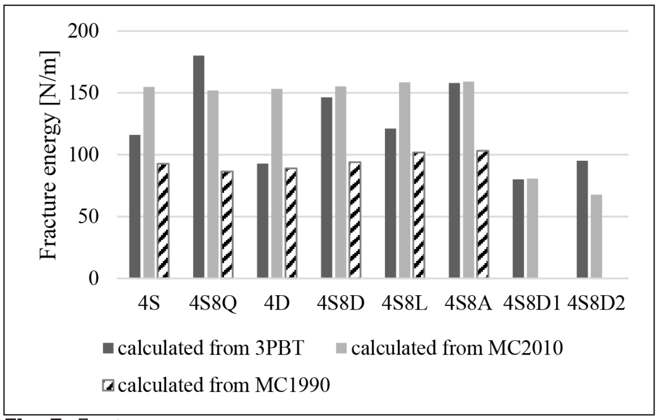


Fig. 7: Fracture energy

Based on the results, it can be said that the fracture energy values calculated by the formula of the Model Code 1990 (Eq. 14.) were close to the values directly calculated by 3PBT result, in case of mixtures with d_{max} =4 mm (the difference was 4-20%), but significantly underestimated them in case of mixtures with d_{max}=8 mm (the difference was 30-50%). The formulas of the *fib* Model Code 2010, on the other hand, overestimated the fracture energy of mixtures with d_{max}=4 mm compared to the values directly calculated by 3PBT (the difference was 35-65%), but gave a good approximation for mixture with d_{max}=8 mm, even for LWCs (the difference was 0.5-25%).

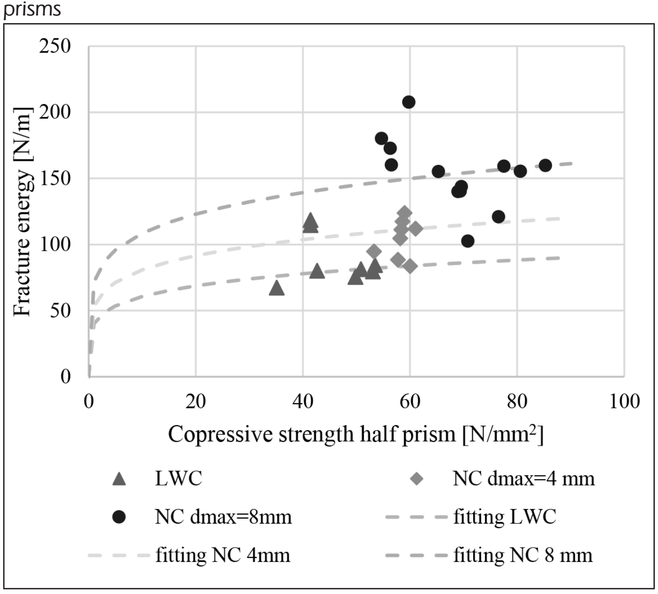
3.6 Fracture energy as a function of compressive strength of half prisms

During the research, compressive strength was also measured on half prisms taken from the prisms previously subjected to 3PBT. The relationship between this compressive strength and thefracture energy is shown in *Fig. 8*.

To describe the relationship between the fracture energy and the compressive strength measured on half prisms, I used the equation of the Model Code 2010 (Eq. 12.) and changed only its coefficients. The measurement results were divided into three groups: NCs with d_{max}=4 mm, NCs with d_{max} =8 mm, LWCs. Equations of the curves fitted to the results are:

$$G_{F,NC,8} = 71.7 f_{cm,0.5prism}^{0.18} \quad (15)$$

Fig. 8: Fracture energy as a function of compressive strength of half prisms



$$G_{F,NC,4} = 53.4 f_{cm,0.5prism}^{0.18} \quad (16)$$

$$G_{F,LWC} = 40.1 f_{cm,0.5prism}^{0.18} \quad (17)$$

where: $G_{F,I}$ fracture energy [N/m],
$f_{cm,0.5prism}$ compressive strength of half prism [N/mm^2].

From the relationship between the measured results and the curves fitted to the results, it can be seen that the *fib* Model Code 2010 formula (Eq. 12.) can be used for NCs with d_{max}=8 mm, if compressive strength measured on half-beams is used instead of compressive strength measured on cubes (coefficient in the original formula is 73; in the case of the curve fitted to the measured value is 71.7). On the other hand, the original formula (Eq. 13.) overestimated the values measured on NCs with d_{max} =4 mm and on LWCs. In these cases, coefficients of the original formula should be changed to 53.4 for NCs with d_{max}=4 mm and to 40.1 for LWCs.

4. CONCLUSIONS

The aim of this paper was to present the calculation methods of the mechanical properties that can be determined from the results of notched 3PBTs. Seven mixtures with different types of aggregates were used (quartz, dolomite, limestone, andesite, expanded clay). To validate the obtained results, I compared the parameters determined from the 3PBTs with the formulas of the *fib* Model Code 2010 (*fib MC2010*, *2013*). Based on the results of the tests, the following can be stated:

- The compressive strength of the concrete mixtures was determined by using two types of specimens: cubes with the size of 150x150x150 mm and half prisms previously subjected to 3PBT. The results show that the individual values measured on cubes and on half prisms were close to each other. Typically, the variance of the values measured on half prisms was larger than that of the values measured on cubes. Based on the value pairs shown, the compressive strength values measured on half prisms were lower than the values measured on cubes if the latter was below 66 N/mm^2. Above 66 N/mm^2 the prisms had higher compressive strength.
- The results showed that the flexural tensile strength values directly calculated by 3PBT and by the compressive strength were close in case of crushed stone aggregate concretes (4S8D, 4S8L, 4S8A), where the difference was 3-10%. In other cases, the values directly calculated by 3PBT were overestimated by the formulae of the Model Code. In case of LWCs, the difference could reach 130-170%. It is important to note that the formulas proposed by the *fib* Model Code 2010 do not correspond to bending tests performed on notched specimens. In case of notches, stress concentrations may change the behaviour of the material. This can be the reason why in case of concrete mixtures which were more sensitive to tension (d_{max}=4 mm, or LWC), the results directly calculated by 3PBT were significantly lower than the results calculated by the *fib* Model Code 2010.
- In case of the results of the modulus of elasticity, the best agreement between the results of the two calculation methods occurred in the case of crushed stone aggregate concretes. For the other mixtures, the difference was significant (95-150%). It is important to note again that in case of a notched specimen, stress concentration can occur, which can change the behaviour of the material, even the elastic behaviour of the concrete in the zone around the crack tip. Therefore, in case of lightweight aggregate and small aggregate size (d_{max}=4 mm), during the determination of the modulus of elasticity, it is recommended to treat the results obtained from 3PBT with caution, and rather to determine the modulus of elasticity by a standard test setup.
- Based on the results, the fracture energy values calculated by the formula of the Model Code 1990 were close to the values directly calculated by 3PBT result, in case of mixtures with d_{max} =4 mm (the difference was 4-20%), but significantly underestimated them in case of mixtures with d_{max}=8 mm (the difference was 30-50%). The formulas of the *fib* Model Code 2010, on the other hand, overestimated the fracture energy of mixtures with d_{max}=4 mm compared to the values directly calculated by 3PBT (the difference was 35-65%), but gave a good approximation for mixture with d_{max}=8 mm, even for LWCs (the difference was 0.5-25%).
- During the research, I also performed compressive strength tests on half prisms previously used for notched 3PBT. Thus, compressive strength values measured on half prisms could be associated with fracture energy values. From the relationship between the measured results and the curves fitted to the results, it could be seen that the *fib* Model Code 2010 formula could be used well for NCs with d_{max}=8 mm. Compressive strength measured on a half prism was used, and the coefficient in the formula was slightly modified. However, the formula of the Model Code overestimated the values measured on NCs with d_{max}=4 mm and on LWC mixtures. In these cases, the coefficient of the original formula had to be significantly modified.

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